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Power corrections to fragmentation functions in non-singlet deep inelastic scattering

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Abstract: We investigate the power-suppressed corrections to the fragmentation functions of the current jet in non-singlet deep inelastic lepton-hadron scattering. The current jet is defined by selecting final-state particles in the current hemisphere in the Breit frame of reference. Our method is based on an analysis of one-loop Feynman graphs containing a massive gluon, which is equivalent to the evaluation of leading infrared renormalon contributions. We find that the leading corrections are proportional to $1/Q^2$, as in e^+e^- annihilation, but their functional forms are different. We give quantitative estimates based on the hypothesis of universal low-energy behaviour of the strong coupling.

Keywords: Deep Inelastic Scattering, QCD, Jets.

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Contents

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1. Introduction

The study of final-state properties in deep inelastic lepton scattering (DIS) has received a great impetus from the increasing quantity and kinematic range of the HERA data. Amongst the most interesting quantities being studied are the fragmentation functions [1], which specify the single-hadron momentum distributions resulting from the fragmentation of the struck parton. Although these functions cannot be calculated using perturbative QCD, their asymptotic scaling violations (logarithmic Q^2 dependence) can be predicted and used to measure the strong coupling α_s, in a similar way to the scaling violations in the deep inelastic structure functions. In addition, the fragmentation functions and their scaling violations can be compared with those measured in other processes, such as e^+e^- annihilation.

One problem with the measurement of α_s using scaling violation, in either structure or fragmentation functions, is that there is Q^2 dependence associated with power-suppressed (higher-twist) contributions, in addition to the dominant logarithmic dependence. These contributions need to be estimated in order to make use of the wide Q^2 coverage of HERA.

Recently so-called ‘renormalon’ or ‘dispersive’ methods of estimating power-suppressed terms have been suggested. By looking at the behaviour of the QCD perturbation series in high orders, one can identify unsummable, factorially divergent sets of contributions (infrared renormalons [2, 3, 4]) which indicate that non-perturbative power-suppressed corrections must be included. The Q^2-dependence of the leading correction to a given quantity can be inferred, and by making further universality assumptions one may also estimate its magnitude. Tests of these ideas provide information on the transition from the perturbative to the non-perturbative regime in QCD. In particular, one can investigate the possibility that an approximately universal low-energy form of the strong coupling may be a useful phenomenological concept [5, 6, 7, 8].

Such an approach has been applied to a wide variety of observables, including DIS structure functions [8, 9, 10, 11, 12], e^+e^- fragmentation functions [13, 14], and event shape variables in e^+e^- annihilation [15, 16, 17, 18, 19, 20, 21] and DIS [22, 23]. Comparisons with experimental data [24, 25, 26, 27] have been encouraging.

Here we extend the same method to DIS fragmentation functions, considering in the present paper the contribution which is non-singlet with respect to the incoming hadron. The singlet part, which involves the gluon distribution function, is formally non-leading in our approach, but may nevertheless be important in the HERA kinematic region, where the gluon density is high. Estimates of power corrections to singlet structure functions have become available very recently [28]; we hope to apply similar techniques to fragmentation functions in a future publication. Meanwhile we concentrate on the non-singlet part, where methods similar to those used for e^+e^- annihilation can be applied. We find that the predicted leading power corrections are proportional to $1/Q^2$, as in e^+e^- annihilation, but their functional forms are different. The hypothesis that power corrections are related to a universal low-energy form for the strong coupling implies that their magnitudes are given by a single non-perturbative parameter. We give quantitative estimates based on the value of this parameter derived from DIS structure function data [9].

In the following section we review the approach of ref. [8]. Section 3 presents the standard leading-order perturbative treatment of DIS fragmentation, which we modify in sect. 4 to estimate the non-singlet power-suppressed corrections using the method outlined in sect. 2. Some numerical results and conclusions are presented in sect. 5.

2. Dispersive estimation of power corrections

We assume that the QCD coupling $\alpha_s(k^2)$ can be defined down to arbitrarily low values of the scale k^2 and that it has reasonable analytic properties, i.e. no singularities other than a cut along the negative real axis. It follows that one can write the formal dispersion relation

$$\alpha_s(k^2) = -\int_0^\infty \frac{d\mu^2}{\mu^2 + k^2}\, \rho_s(\mu^2)\,, \tag{2.1}$$

where the 'spectral function' ρ_s represents the discontinuity across the cut,

$$\rho_s(\mu^2) = \frac{1}{2\pi i}\, \mathrm{Disc}\left\{\alpha_s(-\mu^2)\right\} \equiv \frac{1}{2\pi i}\left\{\alpha_s\left(\mu^2 e^{i\pi}\right) - \alpha_s\left(\mu^2 e^{-i\pi}\right)\right\}. \tag{2.2}$$

We now consider the calculation of some observable F in an "improved one-loop" approximation, i.e. taking into account one-gluon contributions plus those higher-order terms that lead to the running of α_s. As discussed in ref. [8], we expect that

$$F = \alpha_s(0)\mathcal{F}(0) + \int_0^\infty \frac{d\mu^2}{\mu^2}\, \rho_s(\mu^2)\, \mathcal{F}(\mu^2/Q^2)\,, \tag{2.3}$$

where the *characteristic function* $\mathcal{F}(\mu^2/Q^2)$ is obtained by one-loop evaluation of F (divided by α_s) with the gluon mass set equal to μ [3, 15]. The first term on the right-hand side represents the contributions in which a single gluon is produced or exchanged, while the second represents those with more complex final or virtual states, e.g. the 'decay products' of a virtual gluon, which contribute to the running of α_s. In contributions that involve real multi-parton final states, eq. (2.3) with the full spectral function ρ_s in the integrand is obtained only if one sums inclusively over a sufficiently wide class of final states. This point will be discussed more fully in sect. 4.

We can eliminate $\alpha_s(0)$ from eq. (2.3) by means of the dispersion relation (2.1):

$$F = \int_0^\infty \frac{d\mu^2}{\mu^2}\,\rho_s(\mu^2)\,\left[\mathcal{F}(\mu^2/Q^2) - \mathcal{F}(0)\right]. \tag{2.4}$$

Non-perturbative effects at long distances are expected to give rise to a modification in the strong coupling at low scales, $\delta\alpha_s$, which generates a corresponding modification in the spectral function via eq. (2.2):

$$\delta\rho_s(\mu^2) = \frac{1}{2\pi i}\mathrm{Disc}\left\{\delta\alpha_s(-\mu^2)\right\}. \tag{2.5}$$

Inserting this in eq. (2.4) and rotating the integration contour separately in the two terms of the discontinuity, we obtain the following non-perturbative contribution to the observable F:

$$\delta F = \int_0^\infty \frac{d\mu^2}{\mu^2}\,\delta\alpha_s(\mu^2)\,\mathcal{G}(\mu^2/Q^2)\,, \tag{2.6}$$

where, setting $\mu^2/Q^2 = \epsilon$,

$$\mathcal{G}(\epsilon) = -\frac{1}{2\pi i}\mathrm{Disc}\left\{\mathcal{F}(-\epsilon)\right\}. \tag{2.7}$$

Since $\delta\alpha_s(\mu^2)$ is limited to low values of μ^2, the asymptotic behaviour of δF at large Q^2 is controlled by the behaviour of $\mathcal{F}(\epsilon)$ as $\epsilon \to 0$. We see from eq. (2.7) that no terms analytic at $\epsilon = 0$ can contribute to δF. On the other hand for a square-root behaviour at small ϵ,

$$\mathcal{F} \sim a_1\frac{C_F}{2\pi}\sqrt{\epsilon} \qquad \Longrightarrow \qquad \delta F = -\frac{a_1}{\pi}\frac{\mathcal{A}_1}{Q}\,, \tag{2.8}$$

while

$$\mathcal{F} \sim a_2\frac{C_F}{2\pi}\epsilon\ln\epsilon \qquad \Longrightarrow \qquad \delta F = a_2\frac{\mathcal{A}_2}{Q^2}\,, \tag{2.9}$$

where

$$\mathcal{A}_q \equiv \frac{C_F}{2\pi}\int_0^\infty \frac{d\mu^2}{\mu^2}\,\mu^q\,\delta\alpha_s(\mu^2)\,. \tag{2.10}$$

Notice that we express the result (2.6) directly in terms of the modification to the strong coupling itself, rather than that in the derived quantity α_{eff} which was used in some previous publications [8, 9, 20]:

$$\alpha_{\mathrm{eff}}(\mu^2) = \frac{\sin(\pi\mu^2\,d/d\mu^2)}{\pi\mu^2\,d/d\mu^2}\alpha_s(\mu^2) = \alpha_s(\mu^2) - \frac{\pi^2}{6}\left(\mu^2\frac{d}{d\mu^2}\right)^2\alpha_s(\mu^2) + \ldots\,. \tag{2.11}$$

Although α_s and $\alpha_{\rm eff}$ are similar in the perturbative region, they differ substantially at low scales, and the former probably has a simpler behaviour. For example, the even moments of the effective coupling modification,

$$A_{2p} \equiv \frac{C_F}{2\pi}\int_0^\infty \frac{d\mu^2}{\mu^2}\,\mu^{2p}\,\delta\alpha_{\rm eff}(\mu^2)\,, \tag{2.12}$$

have to vanish for all integer values of p, whereas those of $\delta\alpha_s$ do not. The translation dictionary for the moments is in any case rather simple:

$$\mathcal{A}_{2p+1} = (-1)^p\,(p+\tfrac{1}{2})\pi\,A_{2p+1}\,,\qquad \mathcal{A}_{2p} = (-1)^p\,p\,A'_{2p}\,, \tag{2.13}$$

where

$$A'_{2p} \equiv \frac{d}{dp}A_{2p} = \frac{C_F}{2\pi}\int_0^\infty \frac{d\mu^2}{\mu^2}\,\mu^{2p}\,\ln\mu^2\,\delta\alpha_{\rm eff}(\mu^2)\,. \tag{2.14}$$

Studies of power corrections to DIS structure functions suggest that $\mathcal{A}_2 = -A'_2 \simeq 0.2$ GeV^2 [9].

As a clearer representation of the magnitudes of power corrections, we may adopt the approach of refs. [16, 20] and express them directly in terms of moments of α_s over the infrared region. We substitute for $\delta\alpha_s$ in eq. (2.6)

$$\delta\alpha_s(\mu^2) \simeq \alpha_s(\mu^2) - \alpha_s^{\rm PT}(\mu^2)\,, \tag{2.15}$$

where $\alpha_s^{\rm PT}$ represents the expression for α_s corresponding to the part already included in the perturbative prediction. As discussed in ref. [16], if the perturbative calculation is carried out to second order in the $\overline{\rm MS}$ renormalization scheme, with renormalization scale $\mu_{\rm R}^2$, then we have

$$\alpha_s^{\rm PT}(\mu^2) = \alpha_s(\mu_{\rm R}^2) + [b\ln(\mu_{\rm R}^2/\mu^2) + k]\,\alpha_s^2(\mu_{\rm R}^2) \tag{2.16}$$

where for N_f active flavours ($C_A = 3$)

$$b = \frac{11C_A - 2N_f}{12\pi}\,,\qquad k = \frac{(67-3\pi^2)C_A - 10N_f}{36\pi}\,. \tag{2.17}$$

The constant k comes from a change of scheme from $\overline{\rm MS}$ to the more physical scheme [29] in which α_s is preferably defined at low scales. Then above some infrared matching scale $\mu_{\rm I}$ we assume that $\alpha_s(\mu^2)$ and $\alpha_s^{\rm PT}(\mu^2)$ approximately coincide, so that

$$\begin{aligned}\mathcal{A}_2 &\simeq \frac{C_F}{2\pi}\int_0^{\mu_{\rm I}^2} d\mu^2\,\Big(\alpha_s(\mu^2) - \alpha_s(\mu_{\rm R}^2) - [b\ln(\mu_{\rm R}^2/\mu^2) + k]\,\alpha_s^2(\mu_{\rm R}^2)\Big)\\ &= \frac{C_F}{2\pi}\,\mu_{\rm I}^2\,\Big(\bar\alpha_1(\mu_{\rm I}) - \alpha_s(\mu_{\rm R}^2) - [b\ln(\mu_{\rm R}^2/\mu_{\rm I}^2) + k + b]\,\alpha_s^2(\mu_{\rm R}^2)\Big)\,,\end{aligned} \tag{2.18}$$

where

$$\bar\alpha_1(\mu_{\rm I}) \equiv \frac{1}{\mu_{\rm I}^2}\int_0^{\mu_{\rm I}^2} \alpha_s(\mu^2)\,d\mu^2\,. \tag{2.19}$$

The dependence of $\bar{\alpha}_1$ on $\mu_{\rm I}$ is partially compensated by the $\mu_{\rm I}$-dependence of the other terms on the right-hand side of eq. (2.18). The dependence on the renormalization scale $\mu_{\rm R}^2$ should help to compensate the scale dependence of the perturbative part. Notice that if we take $\mu_{\rm R}^2 \propto Q^2$ then $\mathcal{A}_2$ has a logarithmic dependence on Q^2. In general we do expect power corrections to have additional logarithmic Q^2-dependences (anomalous dimensions), but these are probably not given reliably by our 'improved one-loop' approximation.

3. Fragmentation in DIS

A complication in DIS, absent from e^+e^- annihilation, is the presence in the final state of the remnant of the initial-state hadron, i.e. the constituents that did not participate in the hard scattering of the lepton. It is expected that the fragmentation of the remnant will be dominated by soft, non-perturbative physics. While of interest for studying the hadronization process, the remnant fragmentation is not so useful for QCD test, and therefore we concentrate here on aspects of fragmentation that are not sensitive to it. This is conveniently done by looking at the final state in the *Breit frame of reference* [30, 31, 32].

We consider the deep inelastic scattering of a lepton of momentum l from a nucleon of momentum P, with momentum transfer q. The main kinematic variables are $Q^2 = -q^2$, the Bjorken variable $x = Q^2/2P\cdot q$ and $y = P\cdot q/P\cdot l \simeq Q^2/xs$, s being the total c.m. energy squared. Then the Breit frame is the rest-frame of $2xP+q$. In this frame the momentum transfer q is purely spacelike, and we choose to align it along the $+z$ axis:

$$P = \tfrac{1}{2}Q(1/x, 0, 0, -1/x)\,, \quad q = \tfrac{1}{2}Q(0,\ 0, 0, 2)\,. \tag{3.1}$$

To a good approximation, the fragmentation products of the remnant will be moving in directions close to that of the incoming nucleon, i.e. they will remain in the 'remnant hemisphere' H_r ($p_z < 0$). On the other hand the products of the hard lepton scattering will tend to be found in the 'current hemisphere' H_c ($p_z > 0$). In fact in the parton model the scattered parton moves along the current ($+z$) axis with momentum $xP+q = \frac{1}{2}Q(1, 0, 0, 1)$. Thus in the parton model the current hemisphere looks like one hemisphere of the final state in e^+e^- annihilation at centre-of-mass energy Q. Fragmentation studies have shown that this similarity is indeed manifest in hadron spectra and multiplicities [1]. This makes it natural to define the fragmentation functions in terms of particles h appearing in the current hemisphere only, $h \in H_c$.

Since we wish to include only particles in the current hemisphere H_c, we define the fragmentation function F^h for a given hadron species h as a function of the variable $z = 2p_h\cdot q/q^2$, which measures the fraction of the hadron's momentum along the current direction and takes values $0 < z < 1$ in H_c:

$$F^h(z; x, Q^2) = \frac{d^3\sigma^h}{dx dQ^2 dz}\Big/\frac{d^2\sigma}{dx dQ^2}\,. \tag{3.2}$$

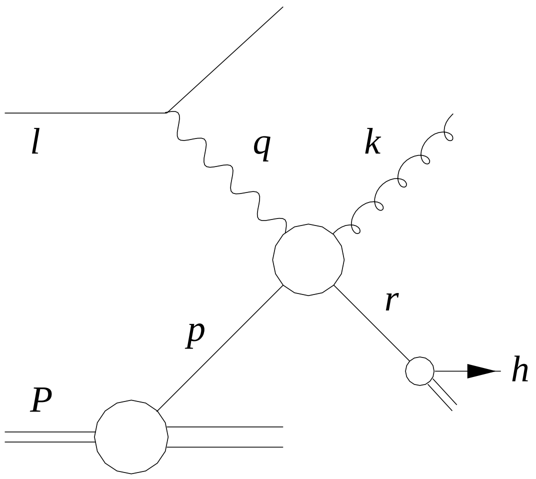

Figure 1: Contribution to jet fragmentation in deep inelastic scattering.

The denominator of this expression is the fully inclusive deep inelastic cross section,

$$\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi\alpha^2}{Q^4}\left\{\left[1+(1-y)^2\right]F_T(x) + 2(1-y)F_L(x)\right\} \tag{3.3}$$

where $F_T(x) = 2F_1(x)$ and $F_L(x) = F_2(x)/x - 2F_1(x)$ are the transverse and longitudinal structure functions (which also have a weak Q^2-dependence that we do not show explicitly). For simplicity we neglect here any contribution from weak interactions (Z^0 or $W^\pm$ exchange). We shall comment on the effect of this in sect. 5. The numerator in eq. (3.2) is the single-hadron inclusive cross section,

$$\frac{d^3\sigma^h}{dxdQ^2dz} = \frac{2\pi\alpha^2}{Q^4}\left\{\left[1+(1-y)^2\right]F_T^h(x,z) + 2(1-y)F_L^h(x,z)\right\} \tag{3.4}$$

where F_T^h and F_L^h are generalized structure functions.

In the parton model (order α_s^0), $F_L = F_L^h = 0$ and

$$\begin{aligned} F_T(x) &= \sum_q e_q^2[q(x) + \bar{q}(x)] \equiv f(x) \\ F_T^h(x,z) &= \sum_q e_q^2[q(x)D_q(z) + \bar{q}(x)D_{\bar{q}}(z)]\,, \end{aligned} \tag{3.5}$$

$q(x)$ and $\bar{q}(x)$ being the quark and antiquark momentum fraction distributions in the target nucleon and $D_q(z)$ and $D_{\bar{q}}(z)$ their fragmentation functions for hadrons of type h.

To first order in α_s, up to two final-state partons can be emitted in the hard lepton-parton subprocess, as illustrated in fig. 1. The momentum of the struck parton is $p = xP/\xi$ $(x < \xi < 1)$ and we define $\eta = P\cdot r/P\cdot q$ $(0 < \eta < 1)$. The parton-level cross section is

$$\frac{d^3\sigma}{dxdQ^2d\eta} = \frac{2\pi\alpha^2}{Q^4}\left\{\left[1+(1-y)^2\right]F_T(x,\eta) + 2(1-y)F_L(x,\eta)\right\} \tag{3.6}$$

where (for $\eta < 1$)

$$F_i(x,\eta) = \frac{\alpha_s}{2\pi}\sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi}\{C_F C_{i,q}(\xi,\eta)[q(x/\xi)+\bar{q}(x/\xi)] + T_R C_{i,g}(\xi,\eta) g(x/\xi)\}\,, \quad (3.7)$$

$C_F = 4/3$, $T_R = 1/2$, and $g(x)$ is the gluon distribution. The coefficient functions are [31]

$$\begin{aligned}
C_{T,q}(\xi,\eta) &= \frac{\xi^2+\eta^2}{(1-\xi)(1-\eta)} + 2\xi\eta + 2 \\
C_{L,q}(\xi,\eta) &= 4\xi\eta \\
C_{T,g}(\xi,\eta) &= \left[\xi^2+(1-\xi)^2\right]\frac{\eta^2+(1-\eta)^2}{\eta(1-\eta)} \\
C_{L,g}(\xi,\eta) &= 8\xi(1-\xi)\,.
\end{aligned} \quad (3.8)$$

In the Breit frame P and q are given by eq. (3.1) and we can write

$$\begin{aligned}
p &= \tfrac{1}{2}Q(1/\xi,0,0,-1/\xi) \\
r &= \tfrac{1}{2}Q(z_0,z_\perp,0,z_3) \\
k &= \tfrac{1}{2}Q(\bar{z}_0,-z_\perp,0,\bar{z}_3)\,,
\end{aligned} \quad (3.9)$$

where

$$\begin{aligned}
z_0 &= 2\eta - 1 + (1-\eta)/\xi \\
z_3 &= 1 - (1-\eta)/\xi \\
\bar{z}_0 &= 1 - 2\eta + \eta/\xi \\
\bar{z}_3 &= 1 - \eta/\xi \\
z_\perp &= 2\sqrt{\eta(1-\eta)(1-\xi)/\xi}\,.
\end{aligned} \quad (3.10)$$

We can distinguish four subregions of phase space, as illustrated in fig. 2:
A: both produced parton momenta k, r in the current hemisphere ($z_3, \bar{z}_3 > 0$);
B: only parton momentum r in the current hemisphere ($z_3 > 0,\ \bar{z}_3 < 0$);
C: only parton momentum k in the current hemisphere ($z_3 < 0,\ \bar{z}_3 > 0$);
D: no produced parton momenta in the current hemisphere ($z_3, \bar{z}_3 < 0$).

The $\mathcal{O}(\alpha_s)$ contributions to the generalized structure functions in eq. (3.4) are of the form

$$\begin{aligned}
F_i^h(x,z) = \frac{\alpha_s}{2\pi}\sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi}\int_0^1 d\eta \\
\times\{C_F C_{i,q}(\xi,\eta) q(x/\xi)[\Theta(z_3)D_q(z/z_3) + \Theta(\bar{z}_3)D_g(z/\bar{z}_3)] \\
+ C_F C_{i,q}(\xi,\eta)\bar{q}(x/\xi)[\Theta(z_3)D_{\bar{q}}(z/z_3) + \Theta(\bar{z}_3)D_g(z/\bar{z}_3)] \\
+ T_R C_{i,g}(\xi,\eta) g(x/\xi)[\Theta(z_3)D_q(z/z_3) + \Theta(\bar{z}_3)D_{\bar{q}}(z/\bar{z}_3)]\}
\end{aligned} \quad (3.11)$$

where D_g is the gluon fragmentation function. In eq. (3.11) the coefficient functions $C_{i,p}$ include virtual corrections at $\eta = 1$, not shown in eqs. (3.8).

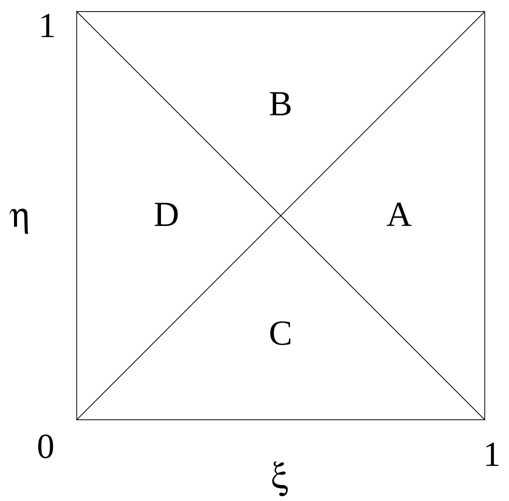

Figure 2: Phase space region for jet production in deep inelastic scattering.

Changing the variable of integration from η to $\zeta = z_3$ or $\bar{z}_3$ as appropriate in each term, we can rewrite this as

$$
\begin{aligned}
F_i^h(x,z) = \sum_q e_q^2 \int_x^1 d\xi \int_z^1 d\zeta \\
&\times [K_{i,qq}(\xi,\zeta) q(x/\xi) D_q(z/\zeta) + K_{i,qg}(\xi,\zeta) q(x/\xi) D_g(z/\zeta) \\
&\quad + K_{i,qq}(\xi,\zeta) \bar{q}(x/\xi) D_{\bar{q}}(z/\zeta) + K_{i,qg}(\xi,\zeta) \bar{q}(x/\xi) D_g(z/\zeta) \\
&\quad + K_{i,gq}(\xi,\zeta) g(x/\xi) D_q(z/\zeta) + K_{i,gq}(\xi,\zeta) g(x/\xi) D_{\bar{q}}(z/\zeta)]
\end{aligned}
\tag{3.12}
$$

where

$$
\begin{aligned}
K_{i,qq}(\xi,\zeta) &= \frac{\alpha_s}{2\pi} C_F C_{i,q}(\xi, 1-\xi+\xi\zeta) \\
K_{i,qg}(\xi,\zeta) &= \frac{\alpha_s}{2\pi} C_F C_{i,q}(\xi, \xi-\xi\zeta) \\
K_{i,gq}(\xi,\zeta) &= \frac{\alpha_s}{2\pi} T_R C_{i,g}(\xi, \xi-\xi\zeta)\,.
\end{aligned}
\tag{3.13}
$$

It is sometimes useful to convert the convolution integrals in eq. (3.12) to simple products by means of a double Mellin transformation,

$$
\tilde{F}(N,M) = \int_0^1 dx \int_0^1 dz\, x^{N-1} z^{M-1} F(x,z)\,,
\tag{3.14}
$$

so that

$$
\tilde{F}_i^h = \sum_q e_q^2 \left(\tilde{K}_{i,qq} \tilde{q} \tilde{D}_q + \tilde{K}_{i,qg} \tilde{q} \tilde{D}_g + \tilde{K}_{i,qq} \tilde{\bar{q}} \tilde{D}_{\bar{q}} + \tilde{K}_{i,qg} \tilde{\bar{q}} \tilde{D}_g + \tilde{K}_{i,gq} \tilde{g} \tilde{D}_q + \tilde{K}_{i,gq} \tilde{g} \tilde{D}_{\bar{q}} \right),
\tag{3.15}
$$

where (note the different powers)

$$
\tilde{K}_{i,qq}(N,M) = \int_0^1 d\xi \int_0^1 d\zeta\, \xi^N \zeta^M K_{i,qq}(\xi,\zeta)\,.
\tag{3.16}
$$

4. Power corrections

To estimate power corrections according to the method outlined in sect. 2, we must recalculate the relevant observables using a non-zero gluon mass-squared $\mu^2 = \epsilon Q^2$, and then extract the terms that are non-analytic at $\epsilon = 0$. In contributions involving an incoming gluon, the "massive" gluon should be treated as an internal line of the singlet process $\gamma^* q \to q'\bar{q}' q$ [28]. Thus the cross section involves two massive gluons and is formally beyond the "improved one-loop" approximation that we are using. Nevertheless such contributions may be important at small x, where the gluon density is large, as is the case at HERA. For the present we avoid these complications by considering the non-singlet contribution. For this we require only the quark coefficient functions, which become

$$
\begin{aligned}
C_{T,q}(\xi,\eta,\epsilon) &= \frac{(1-\eta)(1-\xi)+2\xi\eta(1-\eta)^2-\xi\epsilon}{(1-\eta-\xi\epsilon)^2} + \frac{2\xi\eta(1-\epsilon)}{(1-\eta-\xi\epsilon)(1-\xi)} \\
&\quad + \frac{(1-\eta)(1-\xi)-\xi\epsilon}{(1-\xi)^2} \\
C_{L,q}(\xi,\eta,\epsilon) &= \frac{4\xi\eta(1-\eta)^2}{(1-\eta-\xi\epsilon)^2}
\end{aligned}
\tag{4.1}
$$

in place of those given in eq. (3.8). The kinematic variables that give the momenta according to eq. (3.9) are now

$$
\begin{aligned}
z_0 &= 2\eta - 1 + (1-\eta)/\xi - \epsilon \\
z_3 &= 1 - (1-\eta)/\xi + \epsilon \\
\bar{z}_0 &= 1 - 2\eta + \eta/\xi + \epsilon \\
\bar{z}_3 &= 1 - \eta/\xi - \epsilon \\
z_\perp &= 2\sqrt{\eta(1-\eta)(1-\xi)/\xi - \epsilon\eta}\,.
\end{aligned}
\tag{4.2}
$$

Thus the phase space region is $0 < \eta < 1 - \epsilon\xi/(1-\xi)$, as illustrated in fig. 3. The regions A,…D defined above in terms of the signs of z_3 and $\bar{z}_3$ are as indicated.

The corresponding characteristic functions for the fragmentation functions are given by eq. (3.12) with $K_{i,qq}(x,z)$ etc. replaced by $(C_F/2\pi)\mathcal{K}_{i,qq}(x,z,\epsilon)$ etc., where

$$
\begin{aligned}
\mathcal{K}_{i,qq}(\xi,\zeta,\epsilon) &= \Theta(1-\xi-\epsilon\xi)\,\Theta((1-\xi)(1-\zeta)-\epsilon\xi)\,C_{i,q}(\xi, 1-\xi+\xi\zeta-\epsilon\xi, \epsilon)\,, \\
\mathcal{K}_{i,qg}(\xi,\zeta,\epsilon) &= \Theta(1-\xi-\epsilon\xi)\,\Theta((1-\xi)(1-\xi+\xi\zeta)-\epsilon\xi^2)\,C_{i,q}(\xi, \xi-\xi\zeta-\epsilon\xi, \epsilon)\,.
\end{aligned}
\tag{4.3}
$$

Note that for brevity we have extracted an overall factor of $C_F/2\pi$, which will be absorbed in the non-perturbative factor (2.10).

Neglecting terms that are analytic or less singular than $\epsilon\ln\epsilon$ at $\epsilon = 0$, we find that the Mellin transforms (3.16) are given by

$$
\tilde{\mathcal{K}}_{T,qq}(N,M,\epsilon) = \left[2S_1(N) + 2S_1(M+1) - 3 + \frac{1}{N} + \frac{1}{N+1} + \frac{1}{M+1} + \frac{1}{M+2}\right]\ln\epsilon +
$$

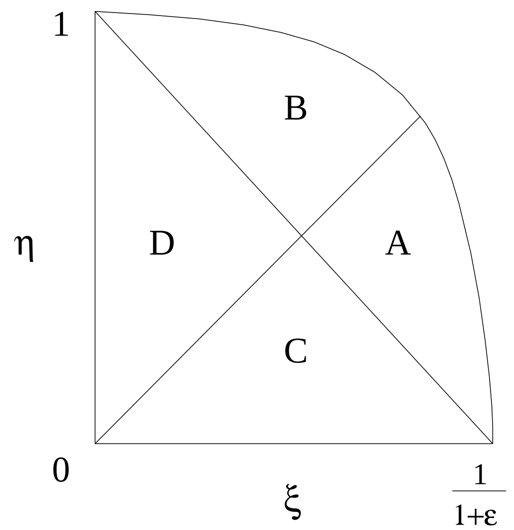

Figure 3: Phase space region with gluon mass-squared $\mu^2 = \epsilon Q^2$.

$$
\begin{aligned}
&\quad + \Bigg[-4S_1(N+1) - 4S_1(M+1) + 6 + 2N + 2M + 2NM \\
&\qquad - \frac{M+2}{N+1} - \frac{4}{N+2} - \frac{N+4}{M+1}\Bigg]\epsilon\ln\epsilon \\
\tilde{\mathcal{K}}_{T,qg}(N,M,\epsilon) &= \left[-\frac{2}{M} + \frac{2}{M+1} - \frac{1}{M+2}\right]\ln\epsilon + \left[1 + \frac{4}{M} - \frac{N+4}{M+1}\right]\epsilon\ln\epsilon \\
\tilde{\mathcal{K}}_{L,qq}(N,M,\epsilon) &= \left[4 - \frac{8}{N+2}\right]\epsilon\ln\epsilon \\
\tilde{\mathcal{K}}_{L,qg}(N,M,\epsilon) &= 0\,,
\end{aligned}
\tag{4.4}
$$

where

$$
S_1(N) = \sum_{j=1}^{N-1}\frac{1}{j}\,. \tag{4.5}
$$

Here we have included the virtual contribution, given in ref. [8].

The expression given above for the gluon fragmentation contribution $\tilde{\mathcal{K}}_{T,qg}$ is valid only for $M > 2$. There is an infrared divergence at $M = 0$, because we integrate the real gluon contribution over one hemisphere only, which does not suffice to cancel the divergent virtual contribution at $\zeta = 0$. For $M = 1$ there is a contribution of $8\sqrt{\epsilon}$ instead of an $\epsilon\ln\epsilon$ term, implying a $1/Q$-correction to this moment of the gluon fragmentation function, as is the case in e^+e^- annihilation [13, 14]. For $M = 2$ the 1 becomes –1 in the coefficient of $\epsilon\ln\epsilon$. All of these changes represent extra contributions at the point $\zeta = 0$, which we can ignore because the fragmentation function at any finite z depends only on the behaviour at $\zeta > z > 0$.

The $\ln\epsilon$ terms in eqs. (4.4) generate the logarithmic scaling violations in the structure and fragmentation functions (see ref. [8]), while the $\epsilon\ln\epsilon$ terms give rise to $1/Q^2$

power corrections, as indicated in eq. (2.9):

$$\begin{aligned}\delta F_i^h(x,z) = \frac{\mathcal{A}_2}{Q^2}\sum_q e_q^2 \int_x^1 d\xi \int_z^1 d\zeta \\ \times[H_{i,qq}(\xi,\zeta)q(x/\xi)D_q(z/\zeta) + H_{i,qg}(\xi,\zeta)q(x/\xi)D_g(z/\zeta) \\ + H_{i,qq}(\xi,\zeta)\bar{q}(x/\xi)D_{\bar{q}}(z/\zeta) + H_{i,qg}(\xi,\zeta)\bar{q}(x/\xi)D_g(z/\zeta)]\,.\end{aligned} \tag{4.6}$$

Inverting the Mellin transforms, eqs. (4.4) give

$$\begin{aligned}H_{T,qq}(\xi,\zeta) &= \left[\frac{4}{(1-\xi)_+} - 2 - 4\xi - +6\delta(1-\xi) + 2\delta'(1-\xi)\right]\delta(1-\zeta) \\ &\quad + \left[\frac{4}{(1-\zeta)_+} - 4 + 2\delta'(1-\zeta)\right]\delta(1-\xi) \\ &\quad -\delta'(1-\xi) - \delta'(1-\zeta) + 2\delta'(1-\xi)\delta'(1-\zeta) \\ H_{L,qq}(\xi,\zeta) &= [-8\xi + 4\delta(1-\xi)]\,\delta(1-\zeta) \\ H_{T,qg}(\xi,\zeta) &= \left[\frac{4}{\zeta} - 4 + \delta(1-\zeta)\right]\delta(1-\xi) + \delta'(1-\xi)\,.\end{aligned} \tag{4.7}$$

Equations (4.6) and (4.7), taken together with the value of $\mathcal{A}_2 \simeq 0.2$ GeV^2 and the parton distribution and fragmentation functions measured in other processes, provide a quantitative estimate of the $1/Q^2$ corrections to current jet fragmentation in non-singlet DIS. Within the context of the dispersive method outlined in sect. 2, the contributions $H_{i,qq}$ from quark fragmentation should be reliable, since they are integrated inclusively over all other parton emission. The gluon contribution $H_{T,qg}$, on the other hand, is less reliably estimated by the "massive gluon" approach adopted here. If, for example, a massive virtual gluon splits into two partons, one of which goes into the remnant hemisphere and one into the current hemisphere, then only the latter will be counted and the full spectral function ρ_s will not be built up in eq. (2.3). The limitations of the massive gluon approach have been studied for $1/Q$ corrections to event shapes in full two-loop order [21, 23], and for $1/Q^2$ corrections to e^+e^- fragmentation functions in the large-N_f limit, i.e. including only quark loops [14]. In the former case a significant but universal enhancement of the power correction was found when going beyond the massive gluon approximation, while in the latter case small corrections were obtained, which could however become larger when gluon loops are included. In both cases the massive gluon estimate of gluon fragmentation effects provided a useful first approximation, and we include it here in the same spirit.

5. Results and conclusions

To obtain some indicative numerical results from eqs. (4.6) and (4.7), we assume for simplicity that the quark fragmentation function D_q is independent of quark flavour and that $D_q = D_{\bar{q}}$. This is reasonable if heavy flavour production is negligible and one sums over fragmentation into all (charged) particles. Then, taking into account the parton-model and non-perturbative contributions, we have from eqs. (3.2)-(3.5) and (4.6)

$$F^h(z;x,Q^2) = D_q(z) + \frac{\mathcal{A}_2}{Q^2}\frac{1}{f(x)}\int_x^1 d\xi \int_z^1 d\zeta\, f(x/\xi)\,\{[H_{T,qq}(\xi,\zeta) \\ -H_{T,q}(\xi)\delta(1-\zeta)]D_q(z/\zeta) + H_{T,qg}(\xi,\zeta)D_g(z/\zeta)\}\,, \tag{5.1}$$

where $f(x)$ is the charge-weighted parton distribution given in eq. (3.5) and $H_{T,q}(\xi)$ is the higher-twist coefficient function for the transverse structure function [8, 9, 10][1]

$$H_{T,q}(\xi) = \frac{4}{(1-\xi)_+} - 2 - 4\xi + 4\delta(1-\xi) + \delta'(1-\xi)\,. \tag{5.2}$$

It should be noted that the integral of $H_{T,qq}(\xi,\zeta)$ with respect to ζ, over the range $0<\zeta<1$, is not equal to $H_{T,q}(\xi)$, because the region $\zeta<0$ makes a contribution of $2\delta(1-\xi)$ to the latter. On the other hand the longitudinal contributions do satisfy the relation

$$H_{L,qq}(\xi,\zeta) = H_{L,q}(\xi)\delta(1-\zeta) \tag{5.3}$$

and therefore no longitudinal higher-twist terms appear in eq. (3.2). Inserting eqs. (4.7) and (5.2) into eq. (5.1), we find

$$F^h(z;x,Q^2) = D_q(z)\left(1 + \frac{\mathcal{A}_2}{Q^2}\,H(z;x)\right)\,, \tag{5.4}$$

where

$$\begin{aligned} H(z;x) = {} & 2xz\frac{f'(x)}{f(x)}\frac{D_q'(z)}{D_q(z)} - x\frac{f'(x)}{f(x)} - 2z\frac{D_q'(z)}{D_q(z)} + \frac{D_g(z)}{D_q(z)} + 2 \\ & + \frac{4}{D_q(z)}\int_z^1 d\zeta\left[\left(\frac{1}{(1-\zeta)_+} - 1\right)D_q\left(\frac{z}{\zeta}\right) + \left(\frac{1}{\zeta} - 1\right)D_g\left(\frac{z}{\zeta}\right)\right] \\ & + x\frac{f'(x)}{f(x)}\frac{1}{D_q(z)}\int_z^1 d\zeta\left[D_q\left(\frac{z}{\zeta}\right) + D_g\left(\frac{z}{\zeta}\right)\right] \\ & + z\frac{D_q'(z)}{D_q(z)}\frac{1}{f(x)}\int_x^1 d\xi\, f\left(\frac{x}{\xi}\right)\,. \end{aligned} \tag{5.5}$$

Figure 4 shows the resulting form of $H(z;x)$ as a function of z for various values of x. We use the ALEPH [33] parametrizations of the light quark and gluon fragmentation functions for charged hadrons at $Q = 22$ GeV, and the corresponding MRST (central gluon) [34] parton distributions. Thus the predictions are at $Q^2 = 484$ GeV^2, but $H(z;x)$ depends only weakly (logarithmically) on Q^2, and in any case our method is not reliable at the level of logarithmic variations. Results become insensitive to x below the values shown in fig. 4. Recall, however, that we have not computed the

[1] Note that the definition here differs from that in refs. [8, 9, 10] by a factor of $-1/\xi$.

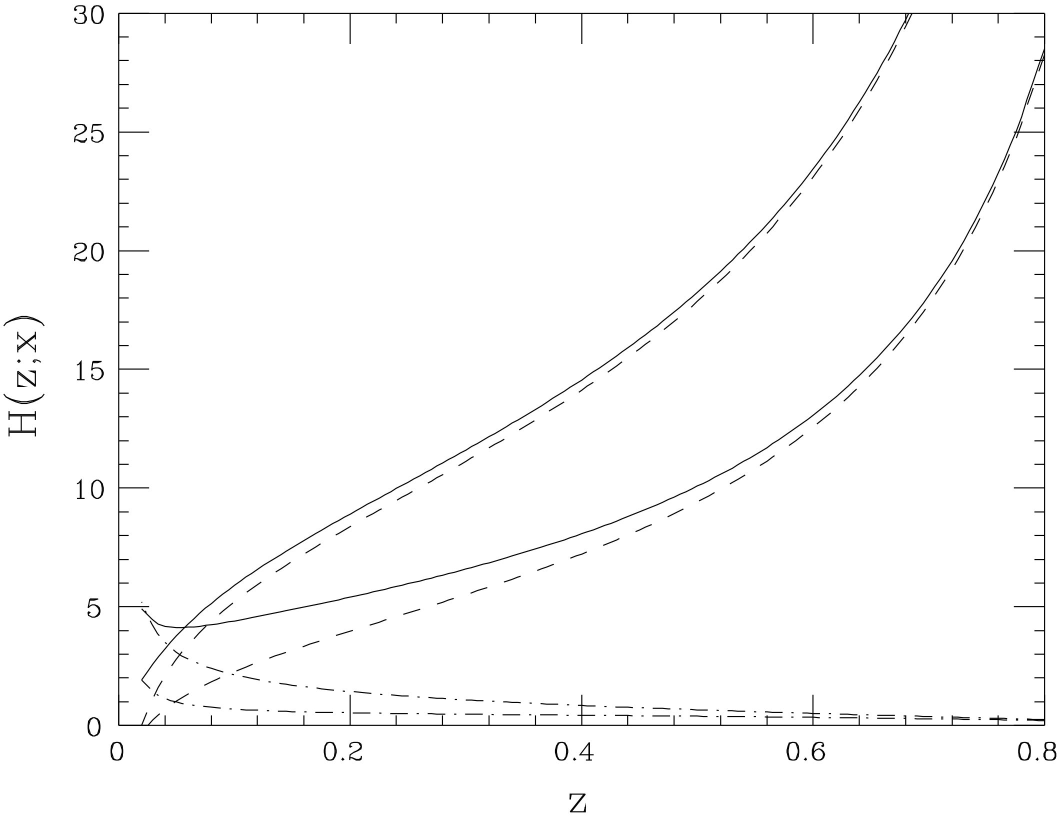

Figure 4: Predicted coefficient of $\mathcal{A}_2/Q^2$ for fragmentation function in non-singlet DIS. Dashed, dot-dashed and solid curves are quark, gluon and total fragmentation. The two sets of curves are for $x = 0.3$ (upper) and 0.1 (lower).

singlet contribution, which may well be important at low x because of the increase in the gluon distribution there.

The predicted power corrections are qualitatively similar to those for fragmentation functions in e^+e^- annihilation [13], though somewhat larger in magnitude. Part of the increase comes from the negative higher-twist correction to the transverse structure function in the denominator of eq. (3.2). The rapid rise in the quark contribution at large x and/or z comes from the product of derivatives in the first term of eq. (5.5). The contribution from gluon fragmentation, although subject to further corrections as discussed earlier, is estimated to be relatively small for $z > 0.2$.

The analysis of e^+e^- fragmentation functions in ref. [33] assumed a $1/Q$ dependence for the power corrections, rather than the $1/Q^2$ dependence predicted here and in ref. [13]. However, the data were mostly obtained at high energies, $Q \simeq M_Z$, where the power corrections are too small to be determined reliably. Indeed the measured $1/Q$ correction was only 1.4 standard deviations from zero, and a $1/Q^2$ dependence was also consistent with the data. For a fully consistent analysis one should extract the fragmentation functions $D_q(z)$ and $D_g(z)$ from e^+e^- data assuming the power corrections predicted in ref. [13], but this would result in only small (technically negligible,

$\mathcal{O}(1/Q^4)$) modifications to fig. 4.

Finally, we comment on the effect of taking weak interactions into account. This introduces contributions from the parity-violating structure function $F_3(x)$ in eq. (3.3) and its analog $F_3^h(x,z)$ in eq. (3.4). However, it was shown in ref. [9] that the predicted higher-twist contributions to F_3 and F_T are the same, i.e. $H_{3,q} = H_{T,q}$ as given in eq. (5.2). Similarly we find that $H_{3,qq} = H_{T,qq}$ and $H_{3,qg} = H_{T,qg}$ as given in eq. (4.7). Hence eq. (5.5) remains valid, provided the parton distribution $f(x)$ is redefined with the appropriate electroweak coefficients in place of the charges in eq. (3.5).

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